Variogram analysis of magnetic data to identify paleochannels of the Omaruru River in Namibia

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Summary

A space domain statistical method for depth estimation from aeromagnetic data is proposed. To demonstrate the method, we have mapped the topography of the crystalline basement underlying the Omaruru Alluvial Plains in Namibia, in order to identify water bearing paleochannels of the Omaruru River.

Introduction

Magnetic maps appear the smoother the greater the depth to the magnetic basement. The smooth/ruggedness of the magnetic field can be quantified by a scaling spectral model [1,2]

$$P(\mathbf{s}) = c_\gamma \, I \, \Theta(\frac{\mathbf{s}}{|\mathbf{s}|}) \, |\mathbf{s}|^{-\gamma} \, e^{-2z|\mathbf{s}|}, \qquad (1)$$

where P(s) is the 2D spectral density ("power spectrum") as a function of the horizontal wavevector s, c_γ is a constant (for fixed γ), I is related to the intensity of magnetization of the basement, $\Theta(\frac{\mathbf{s}}{|\mathbf{s}|})$ reflects the direction of the geomagnetic field, γ is a scaling exponent and z is the depth to the magnetic basement, hence, the parameter of interest.

Previous methods of spectral analysis [3-6] attempt to estimate and interpret the spectral density by transforming a grid of the magnetic data to wavenumber domain. However, the statistical properties are distorted considerably during this procedure. The high wavenumber information is affected by gridding and by edge effects of the Fourier transform (FFT), whereas the low wavenumber portion of the power spectrum is biased towards low wavenumbers by detrending, tapering and radial averaging [6]. Hence the question arises, why one should laboriously transform the measured data to wavenumber domain, when it is possible to transform the spectral model to space domain, instead. The latter can be done analytically, without loss of information. This idea is pursued here.

Variogram analysis

Our aim is to estimate the depth z to the magnetic basement by applying the spectral model (1) to the measured data. As shown in Fig. 1, there are several different ways to accomplish this. To avoid the above mentioned difficulties of estimating the spectral density, it is advantageous to perform the spectral analysis in space domain. To do so, one has to compute a space domain counterpart of the spectral model (1). The first possibility is to use Wiener-Khinchines' formula which relates the spectral density to the auto-correlation function (ACF). However, this rela-

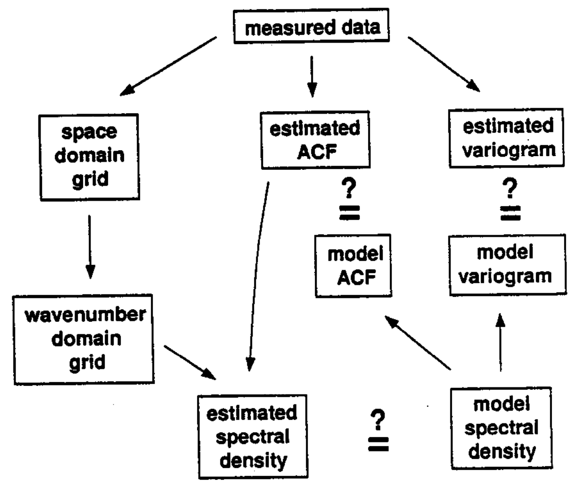

Figure 1: A model spectral density can be used for the interpretation of measured data in several different ways. The variogram approach is the most appropriate for the scaling spectral model (1).

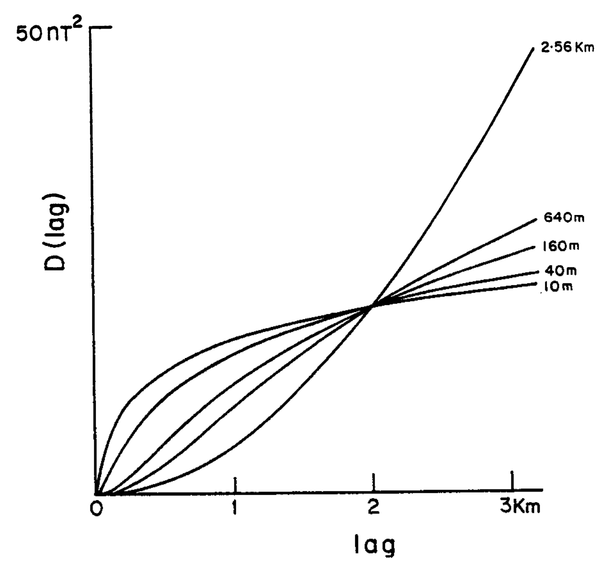

Figure 2: The variogram D is the expected square of the difference between two field values as a function of their spatial separation (lag). Shown here are the model variograms for $\gamma = 2$ and different depths to source.

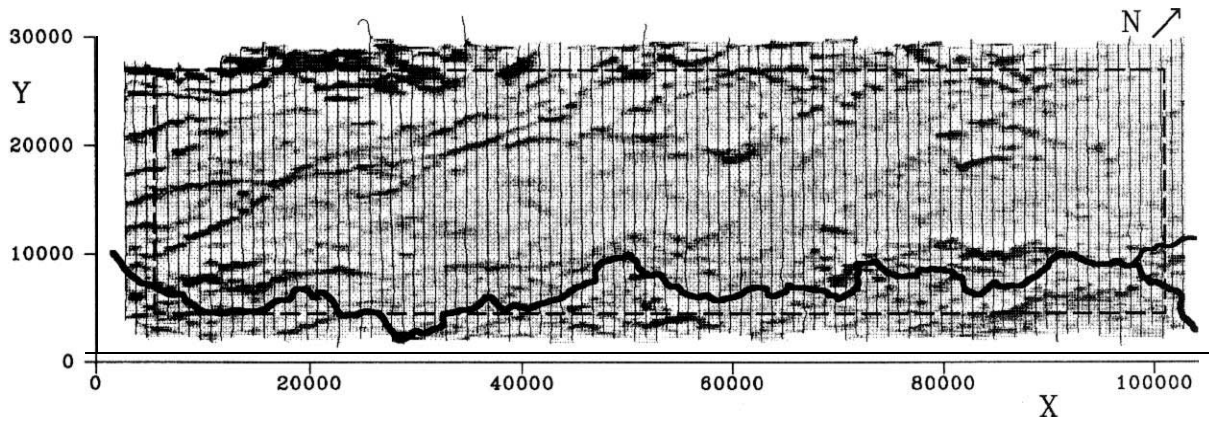

Figure 3: Shaded relief of the anomaly of the total intensity of the magnetic field. The solid line indicates the present bed of the Omaruru. Depth estimates are derived and displayed in Fig. 4 for the area within the dashed rectangle. The coordinates of this rotated system are related to UTM by = xcosa - ysina + 455 000 and UTM_y = xsina + ycosa + 7 570 000 with a = 41.3”.

tionship requires an assumption of stationarity. which is generally not met by magnetic data.

There remains the possibility of using the variogram counterpart of the spectral model. It can be obtained by the integral transform [7, p.435)

$$\mathbf{D}(\mathbf{d}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [1 - \cos(\mathbf{d}\cdot\mathbf{s})]\; P(s)\; ds_x\; da,, \qquad (2)$$

where s and **d** are Z-dimensional vectors and **d• s is** the scalar product of the two. One of the integrals in (2) can be solved analytically by transforming to polar coordinatea. The remaining integral has to be approximated numerically.

Variogram analysis is attractive due to the simple nature of the variogram. The variogram **D(d)** depicts the expected square of the difference between two data values as a function of their spatial separation **d.** A variogram is easier to estimate and interpret than a power spectrum or an auto-correlation function. The effect of the depth to source is illustrated on 1D cross-sections of the 2D variograms in Fig. 2.

As in previous methods of spectral analysis, a window is moved over the survey area to derive consecutive depth values. However, we use the original point located data instead of the gridded data to estimate the variogram of the magnetic field for each window. Each estimated vari-. ogram is then compared with the model variogram (2) to derive the optimum depth to source z within the window. The depth values are finally combined to a topographic map of the crystalline basement.

Example

In line with the agreement on technical co-operation between the Republic of Namibia and the Federal Republic of Germany, an area of 100 km x 26 km along the Omaruru River was surveyed by airborne geophysics [8]. The measured magnetic field is displayed as a shaded relief in Fig. 3. Recalling that the smoother the field, the greater the depth to the basement, a buried valley can be made out, roughly parallel to the present Omaruru. This buried valley is not discernible in the present day topographic map, which is displayed in 3D in Fig. 4a. For Fig. 4b we have subtracted the depth estimates obtained by our variogram analysis from the topographic altitudes. Probable paleochannels of the Omaruru are indicated as dashed lines.

Conclusion

The basement topography of an alluvial plain has been derived by variogram analysis of aeromagnetic data based on a scaling (fractal) spectral model. We claim that this approach offers a considerable improvement over previous methods of spectral analysis, both in terms of accuracy and resolution. This improvement was accomplished by

1. using a scaling spectral model for the basement magnetization, which is more realistic than previous white-noise models
2. using the point-located data and thus avoiding the loss of spectral information during gridding

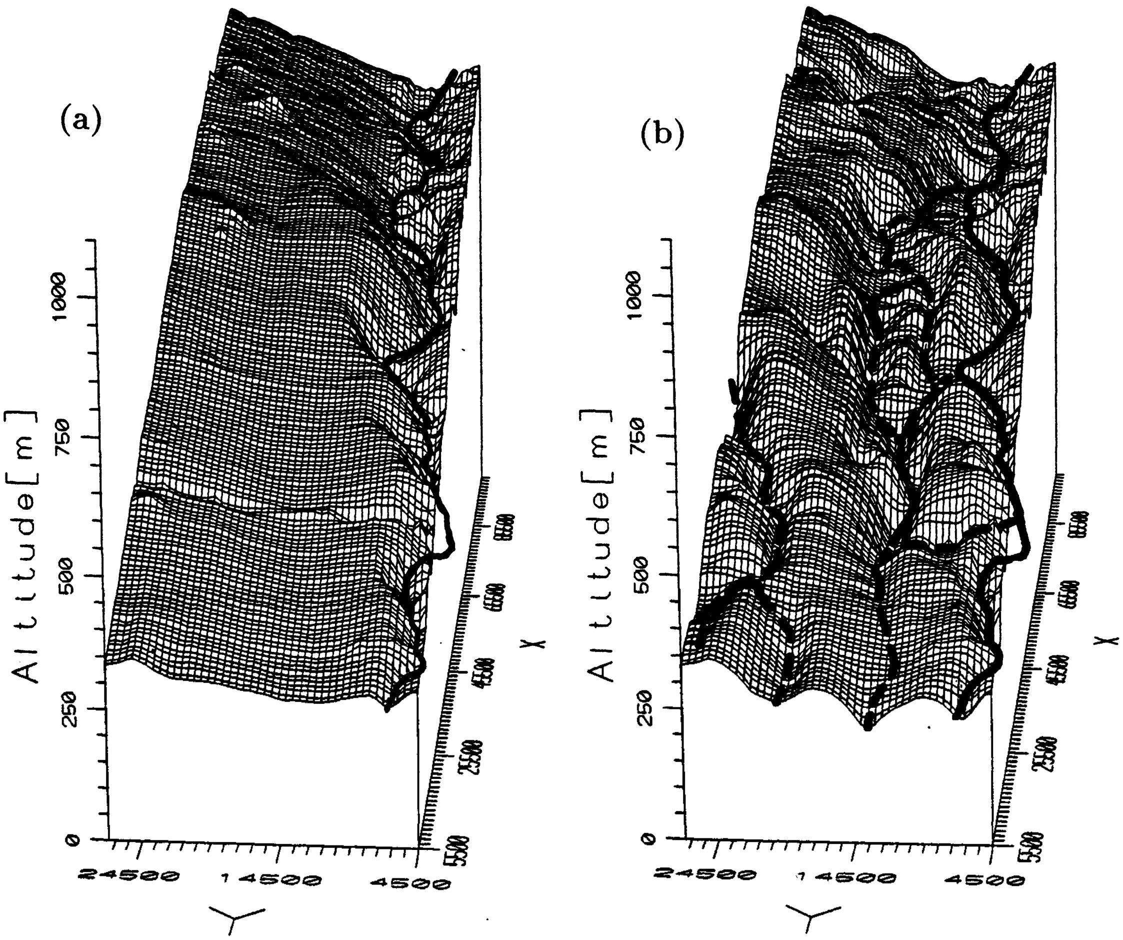

Figure 4: (a) Topography of the survey area. The solid line indicates the present bed of the Omaruru. (b) Topography minus the estimated depth to the crystalline basement. Our interpretation of possible Omaruru paleochannels is given as dashed lines.

3. performing the spectral analysis in space domain and thus avoiding the intricacies of power spectrum estimation.

Acknowledgements

We thank Bernd Röttger and Bernhard Siemon for their participation in the project. Helpful discussions with Derek Fairhead, Alan Reid and Jarin Tulyatid are gratefully acknowledged.

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